Routing optimization with vehicle-customer coordination

Alexandre Jacquillat
Kai Wang
Shuaian Wang
Wei Zhang
Overall research agenda

Air traffic management

On-demand mobility

Logistics decarbonization

Societal applications
Vehicle-customer coordination (VCC)

- “Walking products”: flexibility in pickup/dropoff locations
- How to coordinate vehicles and customers, with which impact?
Coordinated operations

- **Downstream**: smoother vehicle operations
- **Upstream**: how to re-optimize overall routing operations?

→ Which customers to visit, in which sequence, with which vehicles, in which locations and at what times?

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(a) Euclidean distance

(b) Manhattan distance
Ultimate objective: DAR–VCC

- Large-scale optimization formulation
- MISOCO: $\ell_2$-norm
- MILO: $\ell_1$-norm, discrete network
- Weak polyhedral structure due to routing and coordination complexities

\[
Q_{ri}^P = Q_{ri-1}^P + \sum_{j \in \mathcal{P} \cup \mathcal{D}} q_j x_{rij} \quad \forall i \in \mathcal{N}, \forall r \in \mathcal{V}
\]

\[
\overline{T}_j \geq v_{ri} - N_j^V \left(1 - x_{r,i,j+n} \right) \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{P}, \forall r \in \mathcal{V}
\]

\[
\|M_{ri}^V - M_j\| \leq N_j^W \left(1 - x_{rij} \right) \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{P}, \forall r \in \mathcal{V}
\]

\[
\|M_{ri}^V - G_j\| \leq N_j^W \left(1 - x_{rij} \right) \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{D}, \forall r \in \mathcal{V}
\]

\[
v_{ri} \geq \frac{\|M_j - H_j\|}{S_j} - N_j^T \left(1 - x_{rij} \right) \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{P}, \forall r \in \mathcal{V}
\]
**Methodological contributions**

Models and algorithms to optimize routing operations with vehicle-customer coordination in large-scale networks

| Single stop optimization  
| (SSO-VCC) | Core subproblem: Optimization of location and time of a single stop, using geometric insights |
| Multi-stop optimization  
| (MSO-VCC) | Optimization of locations and times of a sequence of stops, using an exact coordinate descent algorithm |
| Vehicle routing (VRP-VCC)  
| Dial-a-ride (DAR-VCC) | Integration of MPO-VCC into routing optimization: subpath-based variables & time-space networks |
| Large-scale online routing optimization | Rolling algorithm for real-world ride-sharing: scalability and win-win-win outcomes |
Single-stop optimization
Single stop optimization: statement

- Operations in the continuous space
- Optimizing stopping time and location
- SOCO with $\ell_2$-norm; LO with $\ell_1$-norm

\[
\begin{align*}
\min \left( v_M + \frac{\|D - M\|}{S} \right), & \quad \text{arrival time} \\
\text{s.t. } v_M & \geq \bar{v} + \frac{\|M - O\|}{S}, \quad \text{vehicle travel time} \\
& \quad \|M - H\| \leq W, \quad \text{customer travel time} \\
& \quad v_M \geq 0, \ M \in \mathbb{R}^2. \quad \text{walking distance} \\
\end{align*}
\]
Single stop optimization: insights

Lemma: the vehicle does not wait at the stopping location

Lemma: (i) straight-path travel; (ii) synchronized arrivals of vehicle and customer; or (iii) binding maximum walking distance

Straight path

Detour

$$v^* = \min_{M \in \mathbb{R}^2} \left\{ \bar{v} + \frac{\|M - O\| + \|D - M\|}{S} : \text{s.t. } \frac{\|M - H\|}{S} \leq \bar{v} + \frac{\|M - O\|}{S}, \|M - H\| \leq W \right\}$$
Single stop optimization: solution

Euclidean space: SSO-VCC can be reduced from a 3D problem (time and space) to a 1D problem (walking angle), solved in $\mathcal{O}(1/\epsilon)$

Manhattan space: SSO-VCC solved via a system of linear equations

Euclidean space

Manhattan space
Multi-stop optimization
Multi-stop optimization: statement

\[
\begin{align*}
    \text{[MSO–VCC]} \quad & \quad \min v_{K+1}, \\
    \text{s.t.} \quad & \quad v_0 = \overline{v}, \ M_0 = O, \ M_{K+1} = D, \\
    & \quad v_k \geq v_{k-1} + \frac{\|M_k - M_{k-1}\|}{S} \quad \forall k = 1, \ldots, K + 1, \\
    & \quad v_k \geq \frac{\|M_k - H_k\|}{S_k} \quad \forall k = 1, \ldots, K, \\
    & \quad \|M_k - H_k\| \leq W_k \quad \forall k = 1, \ldots, K, \\
    & \quad v_k \geq 0, \ M_k \in \mathbb{R}^2 \quad \forall k = 0, \ldots, K + 1.
\end{align*}
\]

Arrival time
Departure time
Vehicle travel time
Customer travel time
Walking distance
Definition domain

Jacquillat—MIT Mobility Forum
Multi-stop optimization: results

- Constrained and non-separable convex optimization problem
  → Coordinate descent algorithm with feasibility and optimality guarantees

- Reformulation as a continuous-space dynamic program
  → A tailored backward induction algorithm (outer loop) that ensures the optimality of all subsequent decisions (inner loop)

\[
\Pi_{k-1}(M_{k-1}, v_{k-1}) = \min_{M_k \in \mathcal{F}_k(M_{k-1}, v_{k-1})} \left\{ \frac{\|M_k - M_{k-1}\|}{S} + \Pi_k \left( M_k, v_{k-1} + \frac{\|M_k - M_{k-1}\|}{S} \right) \right\}
\]

**Theorem:** If \( \|\cdot\| \) is differentiable or the \( \ell_1 \)-norm, MSO–VCC can be solved to optimality by solving SSO–VCC problems iteratively
Multi-stop optimization: algorithm

- Iteratively re-optimize stop $k$, while re-optimizing "tail" $k + 1, \ldots$

Full

$$\min_{M_1 \in \mathbb{F}_1(O, \bar{v})} \left\{ \bar{v} + \frac{\|M_1 - O\|}{S} + \Phi(M_1, \Xi^*(M_1, v_1)) \right\}, \quad \text{where } v_1 = \bar{v} + \frac{\|M_1 - O\|}{S}$$

Algorithm

$$M_1^{(i)} \in \arg\min_{M_1 \in \mathbb{F}_1(O, \bar{v})} \left\{ \bar{v} + \frac{\|M_1 - O\|}{S} + \Phi(M_1, \Xi^*(M_1^{(i-1)}, v_1^{(i-1)})) \right\}, \quad \text{and } v_1^{(i)} = \bar{v} + \frac{\|M_1^{(i)} - O\|}{S}$$
Routing optimization
Routing optimization: statements

- **Vehicle routing problem (VRP–VCC)**
  - Visiting customers with multiple vehicles to minimize travel time

- **Offline dial a ride (DAR–VCC)**
  - Transport customers from origin to destination to maximize profits

- **Online dial-a-ride (O–DAR–VCC)**
  - Dynamic arrivals, re-optimization every 10 s
  - NYT data: 20,000 requests per hour

10 seconds ★ 10 seconds ★ 10 seconds ★ 10 seconds
new customers new customers new customers ★ re-optimization
Subpath-based time-space model

Route-based formulation

Arc-based formulation

Subpath-based formulation
Offline dial-a-ride: subpath model

Legend: ▲ pickup ▼ dropoff

- Node: “empty point” for vehicles
- Arc: customer-carrying trip (subpath)

→ Combination of set partitioning principles and time-space principles

Path optimization [MSO-VCC]  Subpath generation [dynamic program]  Route optimization [time-space network]

\[
\begin{align*}
\max \sum_{u \in U} g^{u} z^{u}, \\
\text{s.t.} \quad \sum_{u \in U} z^{+} - \sum_{u \in U} z^{-} = \begin{cases} 
0 & \forall s \neq s^O, s^D \\
\Lambda & \text{for } s = s^O \\
-\Lambda & \text{for } s = s^D 
\end{cases} \\
\sum_{u \in U} a^{u} z^{u} \leq 1, \forall j \in \mathcal{P}, \\
0 \leq \Lambda \leq |\mathcal{V}|, \\
z^{u} \in \{0,1\}, \forall u \in U.
\end{align*}
\]
## Offline dial-a-ride: sample results

<table>
<thead>
<tr>
<th># customers</th>
<th>CPLEX Solution</th>
<th>CPLEX CPU (s)</th>
<th>Our algorithm Solution</th>
<th>Our algorithm CPU (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>18.3</td>
<td>66</td>
<td>18.3</td>
<td>&lt;1</td>
</tr>
<tr>
<td>7</td>
<td>10.7</td>
<td>&gt;3,600</td>
<td>16.9</td>
<td>&lt;1</td>
</tr>
<tr>
<td>8</td>
<td>9.6</td>
<td>&gt;3,600</td>
<td>24.9</td>
<td>&lt;1</td>
</tr>
<tr>
<td>10</td>
<td>2.2</td>
<td>&gt;3,600</td>
<td>17.7</td>
<td>&lt;1</td>
</tr>
<tr>
<td>15</td>
<td>11.3</td>
<td>&gt;3,600</td>
<td>42.5</td>
<td>&lt;1</td>
</tr>
<tr>
<td>20</td>
<td>7.0</td>
<td>&gt;3,600</td>
<td>41.3</td>
<td>&lt;1</td>
</tr>
<tr>
<td>25</td>
<td>4.6</td>
<td>&gt;3,600</td>
<td>46.4</td>
<td>&lt;1</td>
</tr>
<tr>
<td>50</td>
<td>-</td>
<td>&gt;3,600</td>
<td>63.2</td>
<td>340</td>
</tr>
<tr>
<td>75</td>
<td>-</td>
<td>&gt;3,600</td>
<td>62.6</td>
<td>246</td>
</tr>
<tr>
<td>100</td>
<td>-</td>
<td>&gt;3,600</td>
<td>57.8</td>
<td>180</td>
</tr>
<tr>
<td>125</td>
<td>-</td>
<td>&gt;3,600</td>
<td>49.5</td>
<td>126</td>
</tr>
<tr>
<td>150</td>
<td>-</td>
<td>&gt;3,600</td>
<td>57.6</td>
<td>390</td>
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<td>175</td>
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<td>&gt;3,600</td>
<td>56.9</td>
<td>678</td>
</tr>
<tr>
<td>200</td>
<td>-</td>
<td>&gt;3,600</td>
<td>63.1</td>
<td>2,843</td>
</tr>
</tbody>
</table>

**Small-scale instances:** much faster than CPLEX

**Medium-scale instances:** better and faster than CPLEX

**Large-scale instances:** optimal solution
no solution with CPLEX
Offline dial-a-ride: optimization edge

- Small downstream benefits from vehicle-customer coordination
- Instead, benefits stem from re-optimization of “upstream” routing operations in view of added operating flexibility

| $|\mathcal{V}|$ | No VCC | Fix Seq. + VCC | Fix Cust. + VCC | Our algorithm |
|---|---|---|---|---|
| Profit | % change | Profit | % change | Profit | % change | Profit | % change |
| 1 | 17.4 | (base) | 17.5 | (+0.4%) | 17.5 | (+0.6%) | 23.4 | (+34.4%) |
| 3 | 50.9 | (base) | 51.1 | (+0.5%) | 51.1 | (+0.6%) | 67.5 | (+32.7%) |
| 5 | 82.0 | (base) | 82.4 | (+0.5%) | 82.4 | (+0.5%) | 105.8 | (+29.0%) |
| 10 | 149.3 | (base) | 150.0 | (+0.4%) | 150.1 | (+0.5%) | 188.0 | (+25.9%) |
| 15 | 201.6 | (base) | 201.8 | (+0.1%) | 204.8 | (+1.6%) | 260.0 | (+28.9%) |
| 20 | 247.6 | (base) | 248.0 | (+0.2%) | 253.3 | (+2.3%) | 321.3 | (+29.8%) |
| 25 | 284.0 | (base) | 284.7 | (+0.3%) | 292.7 | (+3.1%) | 369.2 | (+30.0%) |
| 30 | 308.8 | (base) | 310.0 | (+0.4%) | 321.0 | (+4.0%) | 401.7 | (+30.1%) |
Online optimization
Online dial-a-ride: algorithm

- Batching and optimization
- Very large scale: 10,000+ requests/hour
- Dynamic customer arrivals
## Online dial-a-ride: sample results

### Benefits of VCC:
- 6% profit increase (~$40M/year in Manhattan)

### Benefits of optimization:
- Most benefits from re-optimization of upstream vehicle routes, as opposed to downstream adjustments

Pickup flexibility is more beneficial than dropoff flexibility.

### VCC increases customer service and ride-pooling

Profit-sharing opportunities through customer discounts.

### Win-win-win outcomes with profit sharing: profitability (+3%), customer service (+6%), and environmental footprint (-15%)

### Table

<table>
<thead>
<tr>
<th>Metric</th>
<th>No ride-pooling</th>
<th>VCC:PU</th>
<th>No VCC</th>
<th>VCC:PU</th>
<th>VCC:DO</th>
<th>VCC:PU–DO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$137,888</td>
<td>$144,378</td>
<td>$144,662</td>
<td>$152,697</td>
<td>$144,757</td>
<td>$153,714</td>
</tr>
<tr>
<td>Profit increase</td>
<td>(base) 4.8%</td>
<td>(base) 5.6%</td>
<td>0.0%</td>
<td></td>
<td></td>
<td>6.2%</td>
</tr>
<tr>
<td>Revenue increase</td>
<td>(base) 4.4%</td>
<td>(base) 4.9%</td>
<td>0.0%</td>
<td></td>
<td></td>
<td>5.4%</td>
</tr>
<tr>
<td>Cost decrease</td>
<td>(base) 3.6%</td>
<td>(base) 13.1%</td>
<td>2.0%</td>
<td></td>
<td></td>
<td>15.4%</td>
</tr>
<tr>
<td>Downstream contribution</td>
<td>(base) 10.2%</td>
<td>(base) 13.7%</td>
<td>1.2%</td>
<td></td>
<td></td>
<td>14.2%</td>
</tr>
<tr>
<td>Upstream contribution</td>
<td>(base) 89.8%</td>
<td>(base) 86.3%</td>
<td>98.8%</td>
<td></td>
<td></td>
<td>85.8%</td>
</tr>
<tr>
<td>VCC (pickups)</td>
<td>—</td>
<td>9%</td>
<td>13%</td>
<td></td>
<td></td>
<td>13%</td>
</tr>
<tr>
<td>VCC (dropoffs)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
<td>3%</td>
</tr>
<tr>
<td>Pooled trips</td>
<td>—</td>
<td>—</td>
<td>18%</td>
<td>23%</td>
<td>19%</td>
<td>26%</td>
</tr>
<tr>
<td>Acceptance rate</td>
<td>75%</td>
<td>80%</td>
<td>79%</td>
<td>84%</td>
<td>79%</td>
<td>85%</td>
</tr>
<tr>
<td>Max discount</td>
<td>(base) 24.1%</td>
<td>(base) 18.0%</td>
<td>−0.2%</td>
<td></td>
<td></td>
<td>16.0%</td>
</tr>
<tr>
<td>VMT</td>
<td>56,803</td>
<td>54,471</td>
<td>46,705</td>
<td>40,619</td>
<td>45,720</td>
<td>39,612</td>
</tr>
</tbody>
</table>
Models and algorithms to optimize routing operations with vehicle-customer coordination in large-scale networks

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Thank you